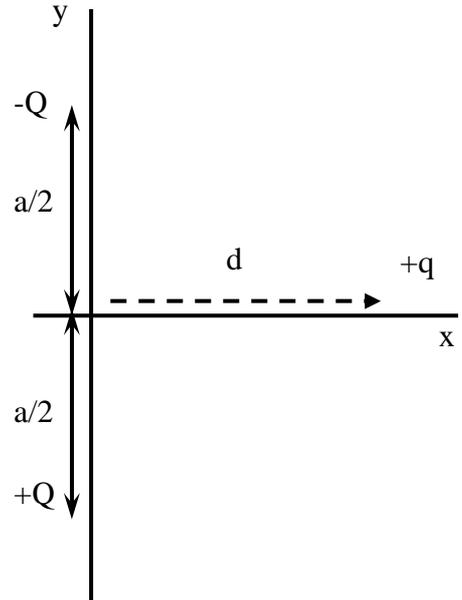


**QUESTION 1**

**[Marks 15]**

(a) Consider the arrangement of point charges shown in the diagram at right where two charges of equal magnitude and opposite sign are positioned on the y-axis equidistant from the origin.



(i) Draw on the diagram the vectors indicating the individual forces on +q from the +Q and -Q charges and the vector indicating the total force on +q due to the +Q and -Q charges. [marks 2]

(ii) Find an expression for the force  $F_q$  on a third charge +q located on the x-axis a distance d from the origin. Feel free to sketch any helpful geometrical construction on the diagram we have provided. [marks 6]

(b) In a few concise sentences explain the meaning of the permittivity  $\epsilon$  in electrostatics. Make reference to Coulomb's law in your answer. [marks 4]

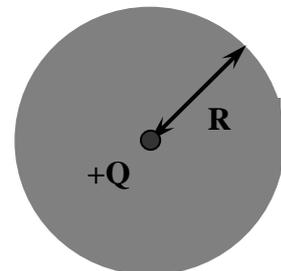
(c) Give a practical or engineering example where  $\epsilon$  has an important role with a brief explanation of the relevant physics. [marks 3]

**QUESTION 2**

**[Marks 16]**

A very simple model describes an atom as a small central nucleus, charge +Q surrounded by a spherical cloud of negative charge, radius R. In this model the volume charge density  $\rho$  out to radius  $r = R$  is given by

$$\rho = \begin{cases} -\frac{5\rho_0}{2} \left[ 1 - \frac{r^2}{R^2} \right] & r \leq R \\ 0 & r > R \end{cases}$$



where  $\rho_0 = \frac{Q}{(4/3\pi R^3)}$

(i) Use Gauss' law to find an expression for the electric field inside and outside the atom [marks 14]

(ii) What is the net charge inside the radial distance  $r = R$ ? [marks 2]

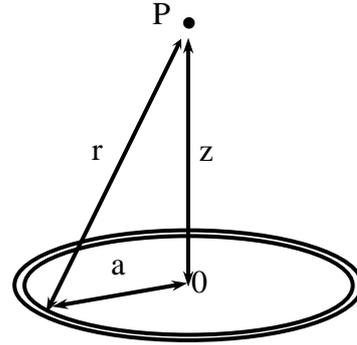
**QUESTION 3**

**[Marks 10]**

The diagram shows a circular ring of uniform electric charge of radius  $a$ . The total charge on the ring is  $Q$  coulombs. The potential at point  $P$  vertically above the centre of the ring is

$$V = k_0 \int \frac{dq}{r}$$

By use of geometry and integration find the expression for  $V$  in terms of  $z$  and  $a$ .



**QUESTION 4**

**[Marks 26]**

- (a) (i) For a 10 m length of copper wire of diameter 0.2 mm and resistance  $0.1 \Omega$ , calculate the drift velocity when a current 100 mA flows in the wire. *[marks 4]*  
[Copper has atomic mass 63.5 u and density  $8,900 \text{ kgm}^{-3}$ ; assume one conduction electron per atom]
- (ii) Comment briefly on the magnitude of the drift velocity value you found in part (i). *[marks 1]*
- (b) (i) The entire length of the copper wire considered in (a) above (the wire must be electrically insulated but assume the insulation thickness is negligible!) is wound into a single-layer coil on an iron rod of diameter 5 mm that serves as a coil-former. Recalling the defining equation for inductance,  $N\Phi = Li$ , estimate the inductance of the coil on the iron former; state explicitly any approximations or assumptions you make. *[marks 4]*
- (ii) Estimate the inductance of a coil with dimensions the same as in (i) above but without the iron former. *[marks 2]*
- (iii) The inductor in (i) with the iron core is to be used as an a.c. circuit component. Give three ways in which this inductor can have characteristics that differ from an ideal or pure inductor providing a brief explanation (1 line in each case) for each. *[marks 3]*  
[For a 'long' solenoid  $B = \mu_0 ni$ . Take the permeability of iron to be  $\mu_{\text{iron}} = 600$ .]
- (c) (i) Sketch the geometry involved in measuring the Hall effect for a thin strip of copper oriented in the  $x$ - $y$  plane with a magnetic induction  $B$  applied in the  $+z$ -direction. Label carefully all current and voltage directions. *[marks 6]*
- (ii) Derive an expression that gives the conduction electron density,  $n$ , in terms of the applied magnetic induction  $B_z$ , the longitudinal current flow, the strip dimensions and the measured Hall voltage. Give all steps in the derivation. [Recall:  $J = I/A = \sigma E$ ,  $I = nAv_d e$ ,  $\mu = e\tau / m_e$ ] *[marks 4]*

- (iii) In a Hall effect measurement, a thin copper strip 1.00 mm wide x 0.01 mm thick and carrying a current of 1.5 A is positioned in a magnetic field  $B=0.1\text{T}$  with the field oriented perpendicular to the wide face of the strip. Find the magnitude of the induced Hall voltage. [marks 2]

**QUESTION 5**

[Marks 21]

- (a) The law of Biot and Savart is

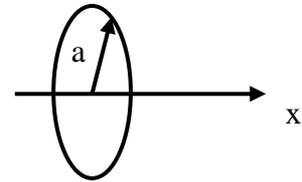
$$d\mathbf{B} = \frac{\mu_0 I}{4\pi} \frac{d\mathbf{l} \times \hat{\mathbf{r}}}{r^2}.$$

Provide a labelled sketch and a brief explanation to describe the meaning of this law. Define all symbols. [marks 4]

- (b)

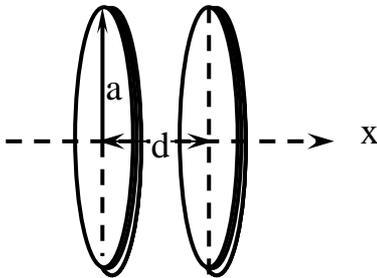
Use the Biot-Savart law to show that the magnetic induction  $B$  on the axis of a current loop of radius  $a$  is given by

$$B = \frac{\mu_0}{2} \frac{Ia^2}{(a^2 + x^2)^{3/2}}$$



where  $x$  is the distance along the axis from the coil centre. [marks 4]

- (c)



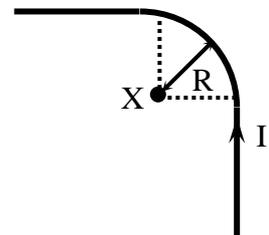
A pair of identical, parallel, circular coils of radius  $a$ , separation  $d$ , and having

$N$  turns each is called a Helmholtz pair. A current  $I$  flows in each coil in the same direction.

- (i) Find an expression in terms of  $a$ ,  $d$ , and  $z$  for the magnetic induction  $B$  on-axis, midway between the coils. [marks 3]
- (ii) Evaluate  $dB_x/dx$  and show that this is equal to zero at the mid-point (half-way between the coils on the  $z$ -axis, choose this to be  $z=0$ ). [marks 2]
- (iii) When  $d=a$  we find that  $d^2B_x/dx^2 = 0$  also. Comment on the usefulness and applications of this. [marks 2]
- (iv) Show that when  $d=a$  the field at  $z=0$  is given by

$$B_{z=0} = \frac{8\mu_0 NI}{5^{3/2} a} \quad \text{[marks 2]}$$

- (d) The diagram below shows a long current-carrying wire bent into a circular  $90^\circ$  arc of radius of  $R$ . The straight sections have equal length. Find the magnetic induction  $B$  due to the whole wire at the point  $X$  located at the centre of the arc, as shown. [marks 4]



**QUESTION 6**

**[Marks 12]**

- (a) Showing all steps in your working, demonstrate (i.e. give a concise account of) how a coil rotated at constant angular frequency  $\omega$  in a magnetic field  $B$  can generate an alternating e.m.f. of constant amplitude and constant frequency given by  $\varepsilon = NAB\omega \sin \omega t$ , where  $A$  is the coil area and  $N$  is the number of turns in the coil. Your account should begin by considering the force on a conduction electron in a moving wire. Provide all necessary equations, giving the meaning of any symbols used, and any fully labelled sketches you need to make a clear explanation. *[marks 6]*
- (b) A rectangular coil of 400 turns with dimensions 6 cm x 4cm rotates at 600 revolution per minute in a magnetic field intensity  $H = 100$  A/m directed perpendicular to its axis of rotation. Using the expression for  $\varepsilon$  shown above calculate the r.m.s. value of the e.m.f. induced in the open-circuit coil. *[marks 4]*
- (c) For the coil described in (b), calculate the open circuit emf directly from Faraday's law. You must show all steps in your workng. *[marks 2]*